Government of **Western Australia**
Curriculum Council

WACE
Western Australian Certificate of Education

Mathematics: Specialist
Formula sheet Units 3C and 3D

Vectors

$|(a,b,c)| = \sqrt{a^2+b^2+c^2}$

$|\mathbf{a}+\mathbf{b}| \leq |\mathbf{a}|+|\mathbf{b}|$

$\mathbf{a}\bullet\mathbf{b} = |\mathbf{a}|\,|\mathbf{b}|\cos\theta = a_1b_1+a_2b_2+a_3b_3$

Vector equation of a line in space:

one point and the slope: $\mathbf{r}=\mathbf{r}_1+\lambda\,\mathbf{l}$

two points: $\mathbf{r}=\mathbf{r}_1+\lambda\,(\mathbf{r}_2-\mathbf{r}_1)$

Cartesian equations of a line in space $\dfrac{x-a}{p} = \dfrac{y-b}{q} = \dfrac{z-c}{r}$

Parametric form of vector equation of a line in space

$x=a+\lambda p$...............(1)

$y=b+\lambda q$...............(2)

$z=c+\lambda r$................(3)

Equation of a plane $\mathbf{r}\bullet\mathbf{n}=c$

$(\mathbf{r}-\mathbf{a})\bullet\mathbf{n}=0$

Trigonometry

In any triangle ABC

$\dfrac{a}{\sin A}=\dfrac{b}{\sin B}=\dfrac{c}{\sin C}$

Area $=\dfrac{1}{2}ab\sin C$

$a^2=b^2+c^2-2bc\cos A$

$\cos A=\dfrac{b^2+c^2-a^2}{2bc}$

In a circle of radius r, for an arc subtending angle θ (radians) at the centre:

Length of arc $= r\theta$ Area of sector $=\dfrac{1}{2}r^2\theta$ Area of segment $=\dfrac{1}{2}r^2(\theta-\sin\theta)$

$\sin(\theta\pm\phi)=\sin\theta\cos\phi \pm \cos\theta\sin\phi$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos(\theta\pm\phi)=\cos\theta\cos\phi \mp \sin\theta\sin\phi$

$\cos 2\theta = \cos^2\theta-\sin^2\theta = 2\cos^2\theta-1 = 1-2\sin^2\theta$

$\tan(\theta\pm\phi)=\dfrac{\tan\theta\pm\tan\phi}{1\mp\tan\theta\tan\phi}$

$\tan 2\theta=\dfrac{2\tan\theta}{1-\tan^2\theta}$

$\lim\limits_{x\to 0}\dfrac{\sin x}{x}=1$

$\lim\limits_{x\to 0}\dfrac{1-\cos x}{x}=0,$

Simple Harmonic Motion: If $\dfrac{d^2x}{dt^2}=-k^2x$ then $x=a\cos(kt+\alpha)=a\sin(kt+\beta)$

Exponentials and logarithms

If $\dfrac{dP}{dt}=kP$, then $P=ae^{kt}$

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Functions

Differentiation

If $f(x)=\sin x$, then $f'(x)=\cos x$

If $f(x)=\cos x$, then $f'(x)=-\sin x$

If $f(x)=\tan x$, then $f'(x)=\sec^2 x=\dfrac{1}{\cos^2 x}$

	Function notation		Leibniz Notation	
	y	y'	y	y'
Product rule	$f(x)\,g(x)$	$f'(x)\,g(x)+f(x)\,g'(x)$	$u\,v$	$\dfrac{du}{dx}v+u\dfrac{dv}{dx}$
Quotient rule	$\dfrac{f(x)}{g(x)}$	$\dfrac{f'(x)\,g(x)-f(x)\,g'(x)}{(g(x))^2}$	$\dfrac{u}{v}$	$\dfrac{\frac{du}{dx}v-u\frac{dv}{dx}}{v^2}$
Chain rule	$f(g(x))$	$f'(g(x))\,g'(x)$	$y=f(u)$ and $u=g(x)$	$\dfrac{dy}{du}\times\dfrac{du}{dx}$

Integration

$\int \sin x\,dx=-\cos x+c$ $\quad\int \cos x\,dx=\sin x+c$ $\quad\int \sec^2 x\,dx=\tan x+c$

Fundamental Theorem of Calculus: $\dfrac{d}{dx}\int_a^x f(t)\,dt=f(x)$ and $\int_a^b f'(x)\,dx=f(b)-f(a)$

Complex numbers

For $z=a+ib$, where $i^2=-1$

Argument: $\arg z=\theta$, where $\tan\theta=\dfrac{b}{a}$ and $-\pi<\theta\le\pi$

Modulus: $\operatorname{mod} z=|z|=|a+ib|=\sqrt{a^2+b^2}=r$

$\arg z_1z_2=\arg z_1+\arg z_2$

Product: $|z_1z_2|=|z_1||z_2|$

$\arg\dfrac{z_1}{z_2}=\arg z_1-\arg z_2$

Polar form: $z=r\operatorname{cis}\theta$, where $r=|z|$ and $\theta=\arg z$

$\operatorname{cis}\theta=\cos\theta+i\sin\theta$ $\quad\operatorname{cis}\theta\operatorname{cis}\phi=\operatorname{cis}(\theta+\phi)$

$\operatorname{cis}(-\theta)=(\operatorname{cis}\theta)^{-1}$ $\quad\operatorname{cis}0=1$

$z_1z_2=r_1r_2\operatorname{cis}(\theta+\phi)$ $\quad\dfrac{z_1}{z_2}=\dfrac{r_1}{r_2}\operatorname{cis}(\theta-\phi)$

For complex conjugates $z=a+ib$ and $\bar{z}=a-ib$,

$\bar{z}=r\operatorname{cis}(-\theta)$ $\quad z\bar{z}=|z|^2$ $\quad\overline{z_1z_2}=\overline{z_1}\,\overline{z_2}$

$\overline{z_1+z_2}=\overline{z_1}+\overline{z_2}$ $\quad z^{-1}=\dfrac{1}{z}=\dfrac{\bar{z}}{|z|^2}$

Matrices

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $|A| = ad - bc$ and $A^{-1} == \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Transformations

Dilation $\begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$

Shear $\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ a & 1 \end{bmatrix}$

Rotation $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

Reflection $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$

Mathematical reasoning

De Moivre's theorem: $(\text{cis}\,\theta)^n = (\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

$z^n = |z|^n \text{ cis }(n\theta)$

$z^{\frac{1}{q}} = r^{\frac{1}{q}}\left(\cos\frac{\theta + 2\pi k}{q} + i\sin\frac{\theta + 2\pi k}{q}\right)$ for $k = 0, 1, 2, \ldots$

Measurement

Trapezium: Area = $\frac{1}{2}(a+b)\times$ height, where a and b are the lengths of the parallel sides

Prism: Volume = Area of base $\times$ height

Cylinder: Total surface area = $2\pi r h + 2\pi r^2$

Volume = $\pi r^2 \times h$

Pyramid: Volume = $\frac{1}{3}$ $\times$ area of base $\times$ height

Cone: Total surface area = $\pi r s + \pi r^2$, s is the slant height

Volume = $\frac{1}{3}\times\pi r^2 \times h$

Sphere: Total surface area = $4\pi r^2$

Volume = $\frac{4}{3}\pi r^3$

Note: Any additional formulas identified by the examination panel as necessary will be included in the body of the particular question.